



Design Principles for Raising Students' Awareness of Implicit Features of Ratio: Creating Opportunities to Make and Catch Mistakes

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Seeing a problem and immediately knowing how to solve it is typically desired in students. However, it can also lead to impulse thinking, where students are unconscious of critical features of concepts and consequently make negligent mistakes. This study investigated the design principles of a secondary mathematics teacher, who designed instructional materials to raise her students' awareness of *implicit features* of ratio and proportionality.

Analysis of her design and implementation revealed two design principles: (1) creating opportunities for students to make mistakes, and (2) sequencing tasks to increase the likelihood for mistakes to happen. These are demonstrated in the following examples, specifically the *catch tasks* located in Question 3 (Figure 1), and Questions 4 and 5 (Figure 2).

To engage students in a more conscious state of thinking, teachers should target common mistakes, misconceptions, and gaps in students' understanding, and incorporate tasks within sequences that will surface these errors by changing the momentum of students' thinking, effectively slowing them down.

Example 1: Without the use of calculator, determine if the following set of ratios are equivalent. Justify your conclusion using the table provided to help you.

| # | Ratio 1 | Ratio 2 | Justification (Show your workings) |
|---|--|---------|--|
| 1 | 21 : 63 | 1 : 3 | Yes / No $\frac{21}{63}$ |
| 2 | 3 : 7 | 24 : 56 | Yes / No $\frac{3}{7}$ |
| 3 | 10 : 3 | 12 : 5 | Yes / No $\frac{10}{3}$ |
| 4 | $2\frac{2}{5} : 1\frac{1}{4}$ Hint: Multiply by a common constant to convert both fractions into integers | 9 : 5 | Yes / No $2\frac{2}{5}$ $1\frac{1}{4}$ |

Figure 1. Example 1 - Determine equivalent ratios

Example 2: Without the use of calculator, express each ratio in the simplest form.

| # | Ratio | Justification (Show your workings) |
|---|---|------------------------------------|
| 1 | 144 : 132 | $\frac{144}{132}$ |
| 2 | $1\frac{1}{2} : 4\frac{1}{2}$ | |
| 3 | $0.48 : 1\frac{1}{5}$ Hint: Convert $1\frac{1}{5}$ to a decimal then simplify from there | |
| 4 | 850g is to 3.4kg | |
| 5 | 1.4 : 7 : 6.3 | |

If two quantities that are **proportional**, they can be expressed in ratio from such as, **a : b = ka :** _____

Figure 2. Example 2 - Simplify ratios